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Crack extension and arrest within the particle subjected to tensile thermoelastic stresses in brittle matrix composites

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A stress relaxation mechanism which entails circumferential cracking around a spherical particle is presented. In a dispersion-strengthened system, composed of spherical particles of different elastic and fracture properties and a matrix of lower thermal expansion, the location and extent of crack propagation is determined by the relative magnitude of elastic and fracture properties of the matrix and the particulate phase. A simple energy balance criteria is adopted to describe the extent of post-initiation crack propagation and to show the relationship between the initial flaw size and the arrested crack length. The major implications of the analysis are discussed in the light of the reported experimental data.

1. Introduction

It has been recognized that the incorporation of particles of different thermoelastic and fracture properties into the brittle matrix may, under certain conditions, lead to significant strengthening of the resultant composite. This has led to the synthesis of a number of dispersion-strengthened composites with various combinations of matrix and particulate phase. Experiments have shown that when suitable choice of matrix and second phase is made, a substantial strengthening was achieved. So far, the extent of strengthening or weakening effects have normally been related to the nature and level of internal stresses generated as a result of the thermal expansion coefficient difference of the phases involved. For example, it has been noticed that the strengthening is achieved only when $\Delta\alpha$ ($=\alpha_m - \alpha_p$, α_m and α_p being the thermal expansion coefficients of the matrix and particle, respectively) approaches zero, or exhibits small negative values. For the systems in this category, Tummala and Friedberg [1] observed strengthening for a glass–ZrO_2 system of $\Delta\alpha = -2.9$ but only when "rounded" particles were incorporated. Similar strengthening was observed in a glass–alumina and glass–zirconia composites [2] exhibiting negative $\Delta\alpha$ values. Some strengthening has also been observed in a glass–tungsten system of near zero $\Delta\alpha$ [3].

In other instances, it has been found that, if the negative value of $\Delta\alpha$ is too large, the thermoelastic stress induces decohesion at the interface or microcracking either in the particle or in the matrix. This has been encountered in a glass–nickel system for $\Delta\alpha = -12$ [4, 5]. Weakening has also been observed in an Si_3N_4–SiC system of negative $\Delta\alpha$ [6].

The next system of interest is the glass–thoria composite in which some form of strengthening has been observed for a small negative value of $\Delta\alpha$ [7].

The object of the present paper is to identify factors controlling the microcracking condition and the location of crack initiation in two-phase systems in the presence of small tensile hydrostatic stress in the particle developed as a result of the thermal expansion difference. For the present analysis a model system is selected which consists of spherical particles dispersed in a brittle matrix of lower thermal expansion (negative $\Delta\alpha$).

2. Microcracking condition

For elastically isotropic brittle matrix–ductile particle composites ($E_m = E_p$, $\nu_m = \nu_p$), it has been shown that the location of microcrack initiation and, therefore, stress relaxation process is governed by the interfacial properties and the strength of the particulate and matrix phases [8]. Bearing in mind the fact that most of dispersion-strengthed systems consist of a brittle matrix and spherical particles of different elastic properties, it is of interest to examine the effect that such particles have on crack-propagation behaviour and overall strength of the resultant composite. Thus, the following analysis is concerned with the mechanism of stress relaxation and post-initiation crack propagation in a dispersed composite containing spherical particles of different elastic properties and the linear thermal expansion coefficient higher than that of the matrix. The system is assumed to contain pre-existing cracks of circular shape, uniformly distributed and non-interacting (Fig. 1). Furthermore, it is assumed that crack extension under differential thermal contraction occurs instantaneously and corresponds to the "fixed grip" condition [9]. According to Berry [10], the extension of cracks not only lowers the Young's modulus but also relaxes the residual stress. The effect of cracks on Young's modulus is given by the expression [11]

$$E = E_0 \left[1 + \frac{16(1-\nu^2)NC^3}{9(1-2\nu)}\right]^{-1} \tag{1}$$

where E_0 is the Young's modulus of crack-free material, C is the crack length, N is the number of cracks per unit volume, and ν is the Poisson's ratio.

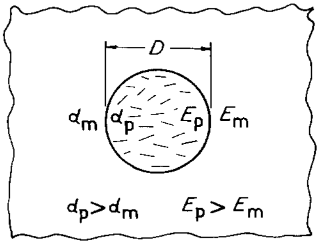

Figure 1 Schematic illustration of a particle embedded in a matrix of lower thermal expansion.

On cooling from high temperature, crack extension may occur in the particle, at the particle–matrix interface or in the matrix, but for the sake of the present analysis, it will be assumed that cracks occur first in the particle. The energy per unit volume of a spherical particle subjected to uniform thermoelastic stress is [7]:

$$W_p = 3(\Delta\alpha\Delta T)^2(1-2\nu_p) \times \left\{2E_p\left[\frac{1+\nu_m}{2E_m} + \frac{1-2\nu_p}{E_p}\right]^2\right\}^{-1} \tag{2}$$

If N cracks per unit volume of a particle are formed on cracking, the energy per unit volume of the particle is expressed as the sum of the elastic energy stored and the surface energy of cracks:

$$W_t = -\frac{3(1-2\nu_p)(\Delta\alpha\Delta T)^2}{2E_{op}} \times \left[1 + \frac{16(1-\nu_p^2)NC^3}{9(1-2\nu_p)}\right]\left\{\frac{1+\nu_m}{2E_{om}} + \frac{1-2\nu_p}{E_{op}}\left[1 + \frac{16(1-\nu_p^2)NC^3}{9(1-2\nu_p)}\right]\right\}^{-2} + 2\gamma_p N\pi C^2 \tag{3}$$

where E_{op} and E_{om} are the Young's moduli of the crack-free particle and matrix, respectively, and γ_p is the fracture surface energy of the particle.

According to Griffith's theory, the crack extension occurs when the following condition is fulfilled:

$$\mathrm{d}W_t/\mathrm{d}C = 0 \tag{4}$$

On substituting Equation 3 into Equation 4 and differentiating, gives the expression for the minimum differential strain required for crack extention:

$$\Delta\alpha\Delta T = \left[\frac{\gamma_p E_{op}\pi}{2(1-\nu_p^2)C}\right]^{1/2}\left\{\frac{1+\nu_m}{2E_{om}} + \frac{1-2\nu_p}{E_{op}}\left[1 + \frac{16(1-\nu_p^2)NC^3}{9(1-2\nu_p)}\right]\right\}^{3/2} \times \left\{\frac{1+\nu_m}{2E_{om}} - \frac{1-2\nu_p}{E_{op}}\left[1 + \frac{16(1-\nu_p^2)NC^3}{9(1-2\nu_p)}\right]\right\}^{-1/2} \tag{5}$$

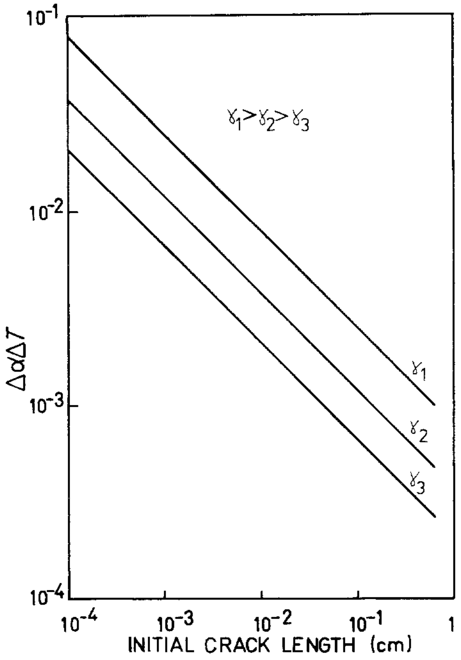

Figure 2 Critical differential strain required to initiate crack propagation in the particle as a function of initial crack length and fracture toughness of the second phase.

If, however, cracking occurs at the particle–matrix interface, certain parameters in Equation 5 must be corrected to include the effect of boundary phase.

For short initial cracks, $[16(1-\nu_p^2)NC^3/9(1-2\nu_p)] \ll 1$, Equation 5 simplifies to:

$$\Delta\alpha\Delta T = \left[\frac{\pi\gamma_p E_{op}}{2(1-\nu_p^2)C}\right]^{1/2}\left[\frac{1+\nu_m}{2E_{om}}+\frac{1-2\nu_p}{E_{op}}\right]^{3/2} \times \left[\frac{1+\nu_m}{2E_{om}}-\frac{1-2\nu_p}{E_{op}}\right]^{-1/2} \tag{6}$$

Fig. 2 illustrates the variation of minimum differential strain required for crack initiation with initial crack length for the fracture energy indicated.

It is evident from Equation 6 that the differential strain required for crack initiation is related to the length of pre-existing flaw (C) and must, therefore, be a function of strength of particulate phase. For a circular crack of length $2C$, the critical fracture stress, σ_{fp}, is [12]

$$\sigma_{fp} = [\gamma_p E_{op}\pi/2C(1-\nu_p^2)]^{1/2} \tag{7}$$

Combining Equations 6 and 7 gives (for short initial crack length):

$$\Delta\alpha\Delta T = \sigma_{fp}\left[\frac{1+\nu_m}{2E_{om}}+\frac{1-2\nu_p}{E_{op}}\right]^{3/2} \times \left[\frac{1+\nu_m}{2E_{om}}-\frac{1-2\nu_p}{E_{op}}\right]^{-1/2} \tag{8}$$

Equation 8 predicts that the critical mismatch strain required to initiate particle cracks is directly related to the strength level of the particle under pure hydrostatic tension. Under such stress, the particle will behave as an ideal linear elastic material because the hydrostatic nature of the stress within the particle favours cleavage crack initiation and suppresses plastic deformation [8].

In developing Equation 8, it has been assumed that crack extension occurs in the particle only. However, the location of crack extension depends on the relative magnitude of critical mismatch strain of matrix and particle as predicted by Equation 8. For ideally linear elastic material, the critical fracture strain of the matrix under uniaxial tension is

$$\epsilon_m = \frac{\sigma_{fm}}{E_m} \tag{9}$$

where σ_{fm} is the fracture stress, and E_m is the Young's modulus of the matrix.

Clearly, particle, not matrix cracking will occur only if the critical fracture strain of the matrix is larger than the critical fracture strain of the particle, i.e.

$$\frac{\sigma_{fm}}{E_{om}} > \sigma_{fp}\frac{\phi}{2E_{om}E_{op}} \tag{10}$$

or

$$\sigma_{fm} > \sigma_{fp}\frac{\phi}{2E_{op}} \tag{11}$$

where

$$\phi = \left\{\frac{[E_{op}(1+\nu_m)+2E_{om}(1-2\nu_p)]^3}{E_{op}(1+\nu_m)-2E_{om}(1-2\nu_p)}\right\}^{1/2}$$

If Equation 11 is not satisfied, the first microcracks will appear in the matrix material, thereby forming so-called circumferential cracks (Fig. 3). As Equation 11 shows, the overall strength of the composite is governed by the strength and elastic properties of the matrix and particle. The condition of microcrack extension as defined by Equation 11 is graphically illustrated in Fig. 4.

So far, discussion has been concentrated on the condition of crack initiation. After initiation, the extent of crack propagation before arrest is governed by the kinetic energy of the crack. Owing to kinetic energy, the crack will propagate

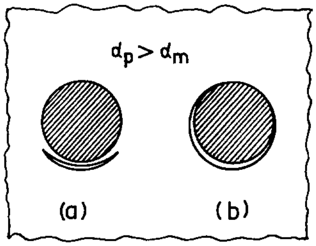

Figure 3 Circumferential microcrack formation around a spherical particle subjected to tensile thermoelastic stress (negative $\Delta\alpha$): (a) partial circumferential cracking; (b) complete circumferential cracking.

until the potential energy released equals the total surface fracture energy of cracks. The condition of crack arrest at some final crack length, C_f, can be obtained from the expression [8–11]:

$$\begin{aligned}&\frac{3(\Delta\alpha\Delta T)^2(1-2\nu_p)}{2E_{op}}\left\{\left[1+\frac{16(1-\nu_p^2)NC^3}{9(1-2\nu_p)}\right]\right.\\&\times\left\{\frac{1+\nu_m}{2E_{om}}+\frac{1-2\nu_p}{E_{op}}\left[1+\frac{16(1-\nu_p^2)NC^3}{9(1-2\nu_p)}\right]\right\}^{-2}\\&-\left[1+\frac{16(1-\nu_p^2)NC_f^3}{9(1-2\nu_p)}\right]\left\{\frac{1+\nu_m}{2E_{om}}+\frac{1-2\nu_p}{E_{op}}\right.\\&\left.\left.\times\left[1+\frac{16(1-\nu_p^2)NC_f^3}{9(1-2\nu_p)}\right]\right\}^{-2}\right\}=2\pi\gamma_p N(C_f^2-C^2)\end{aligned}\qquad(12)$$

where C is the initial crack length. For short initial crack length, $C_f \gg C$, the final crack length is obtained by substituting Equation 5 into Equation 12:

$$\begin{aligned}C_f=&\left\{3(1-2\nu_p)\left[\frac{1+\nu_m}{2E_{om}}+\frac{1-2\nu_p}{E_{op}}\right]\right\}^{1/2}\\&\times\left\{8(1-\nu_p^2)NC\left[\frac{1+\nu_m}{2E_{om}}-\frac{1-2\nu_p}{E_{op}}\right]\right\}^{-1/2}\end{aligned}\qquad(13)$$

Equation 13 indicates that the extent of crack propagation in the particulate phase is a function of the elastic properties of the materials, crack density and the initial crack length. It should be noted, however, that if cracking occurs in the particle, the size of a final crack length will eventually be limited by the particle size.

If C_f is assumed to be the largest crack in the specimen, then the strength of the composite can be obtained by combining Equations 7 and 13 (for $C = C_f$):

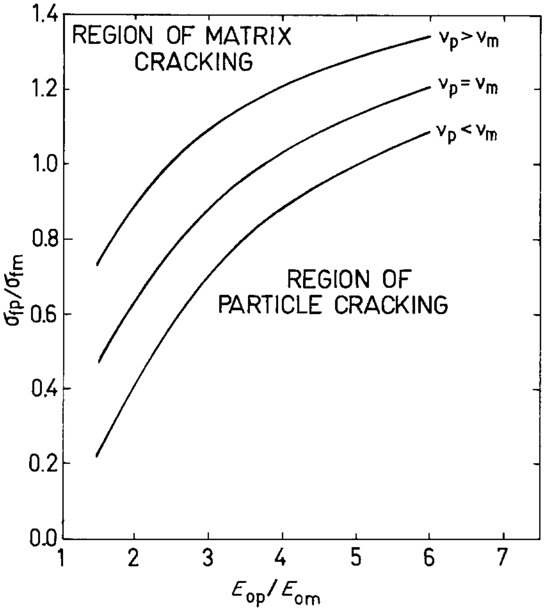

Figure 4 The effect of elastic properties and strength of the matrix and particulate phase on circumferential cracking.

$$\begin{aligned}\sigma_f=&\left[\frac{E\gamma\pi}{2(1-\nu^2)}\right]^{1/2}\\&\times\left\{8(1-\nu_p^2)NC\left[\frac{1+\nu_m}{2E_{om}}-\frac{1-2\nu_p}{E_{op}}\right]\right\}^{1/4}\\&\times\left\{3(1-2\nu_p)\left[\frac{1+\nu_m}{2E_{om}}+\frac{1-2\nu_p}{E_{op}}\right]\right\}^{-1/4}\end{aligned}\qquad(14)$$

where γ, E and ν represent the fracture surface energy, Young's modulus and Poisson's ratio, respectively, either of the particle, matrix or boundary phase, depending on whether the crack extension occurs in the particle, in the matrix or at the particle–matrix interface. If cracking occurs at the particle–matrix interface or in the matrix, experiments have shown that such cracks finally attain circumferential or arc-shaped form as shown in Fig. 3.

Specifically, Equation 14 applies to the cases where the size of a circumferential crack is smaller than the interparticle spacing, i.e. when the interaction between the crack tip stress field and the thermoelastic stress field of neighbouring particles is negligible. For a small mean free path of second-phase composites where the interaction of neighbouring stress fields is significant, the strength of the composite is not determined only by the energetics of the process, but also by the interparticle spacing. The strength behaviour of dispersed composites with high volume fraction of the second phase has been treated by Hasselman and Fulrath [4] and will not be considered here.

3. Discussion

From the solution of thermoelastic stress concentration around a spherical particle embedded in a brittle matrix of lower thermal expansion, it is known that the tangential component of the stress acts to close all radial cracks emanating from the particle–matrix interface [13]. The radial component, on the other hand, tends to open circumferential cracks located either in the matrix or in the particle. As defined by Equation 11, the location of cracks responsible for fracture of the dispersed composite is determined by the strength level and elastic constants of the particle and matrix phases, as illustrated in Fig. 4. It is evident from Fig. 4 that in two-phase systems where both phases are of comparable strength, the particle cracking will occur when the elastic modulus of the particle is appreciably higher than that of matrix phase. When Equation 11 is applied to a well-studied glass–thoria system [7], on substituting pertinent values for $E_p = 25 \times 10^4$ MPa, $\nu_p = 0.275$, $E_m = 7 \times 10^4$ MPa and $\nu_m = 0.20$, it shows that to avoid matrix cracking, the glass matrix must possess a strength greater than 90% of the strength of the thoria particle. The measured strength of thoria is approximately 97 MPa [14], whereas the strength of glass is 48 to 100 MPa [14]. From Equation 11 it follows that, in order to avoid matrix cracking in a glass–thoria composite, the tensile strength of the glass must be in excess of 86 MPa.

If, however, instead of thoria, alumina particles with an average tensile strength of 250 MPa [14] and a Young's modulus $E_p = 42 \times 10^4$ MPa, are incorporated in a glass matrix of the same elastic properties, the strength of the glass matrix should be in excess of 215 MPa in order to suppress its cracking.

Similarly, when other crystalline particles of higher strength, such as tungsten, TiO_2 and ZrO_2, are dispersed in a glass matrix of lower thermal expansion, very large differential strains are required to induce particle cracking. However, if lower strength particles are incorporated in a higher strength matrix, such as SiC dispersed in Si_3N_4 matrix, differential strains of the order of 0.002 26 are required before cracks are initiated in the SiC particle. The critical fracture strain of Si_3N_4 under uniaxial tension is approximately 0.001 33, which is much smaller than the critical differential strain required for crack extension in the SiC particle. Although the tensile strength of SiC ($\sigma_{fp} = 310$ MPa) is significantly smaller than the tensile strength of Si_3N_4 matrix ($\sigma_{fm} = 410$ MPa) [14], particle cracking is not likely to occur in this composite.

The above results indicate that, in a majority of conventionally made composites, microscopic fracture initiates at matrix cracks and the matrix phase is the phase that controls the overall strength of the system. The incorporation of high-strength particles in a lower strength matrix does not necessarily lead to significant strengthening of the resultant composite. Noticeable strengthening can be achieved in a residual stress-free matrix when the volume fraction of the dispersed phase is sufficiently high to cause reduction of pre-existing crack length [4]. It follows, therefore, that, for a low volume fraction of second-phase composites, in which the interparticle spacing is larger than the inherent flaw size of the matrix, the presence of the dispersed phase is expected to have no effect on the composite strength. A typical example of this is found in a glass–alumina system in which no volume fraction effect was observed up to the point when interparticle spacing becomes comparable to the inherent flaw size of the matrix [4].

As shown by Equations 6 and 8, crack extension occurs when a critical strain for fracture initiation is reached. The location of crack extension is determined by the strength and the elastic properties of the phases involved. If cracking occurs within the particle, cracks will have a circular shape and will extend until their length reaches the particle diameter. The strength of such a composite is expected to be governed by the particle size, provided that the particle size is lower than the pre-existing flaw size responsible for fracture. When cracking occurs at the particle–matrix interface, or in the matrix, the length of an arrested crack is not limited by the particle diameter, but owing to its circumferential form its maximum length cannot exceed the particle circumference by any great amount. In such a case, the microscopic fracture is expected to start from "pseudo" pores generated by circumferential cracking, and the strength of the composite should obey a modified Griffith equation.[*] A typical example of this is found in a glass–thoria system of large negative $\Delta\alpha$ values [7]. Based on this consideration, a logical conclusion would be that, if a brittle matrix contains spherical particles with a maximum diameter smaller than the pre-existing flaw size of the matrix, the matrix will be the phase that controls the overall strength of the composite.

An important consequence that can be inferred from the present analysis is that, in a composite of negative $\Delta\alpha$, the size of the particulate phase is more important than the magnitude of the residual stress itself, because the residual stress will be relaxed during crack extension, leaving "pseudo" pores which normally serve as fracture precursors. From the strengthening point of view, the most advantageous composite is that consisting of a brittle matrix of lower thermal expansion and uniformly distributed spherical particles of sizes smaller than the pre-existing flaw size of the matrix.

Perhaps the most significant support for the stress relaxation concept advanced in the previous paper [8] and extended in the present work, may be found in the composites in which the residual stress is many times higher than both the matrix and the particle strength. Such composites should exhibit no strength and are expected to fracture on cooling from the fabrication temperature. The fact that these composites possess considerable residual strength, serves as an indication that the stress relaxation mechanism, occurring simultaneously with crack extension, must be present. The significance of the residual stress relaxation phenomenon in strengthening brittle ceramics has been investigated by Tree *et al.* [15].

Clearly, in the composites in which the residual stress remains unrelaxed, the effect of these stresses must be included in the equation for strength [16].

4. Conclusions

The location of crack extension and arrest conditions within a well-bonded spherical particle of different elastic properties dispersed in a brittle matrix of lower thermal expansion, can be predicted using a simple energy balance criteria. Crack extension, followed by the residual stress relaxation, can occur in the particle, at the particle–matrix interface or in the matrix depending on the interfacial properties, the tensile strength of particulate and matrix phase and on their elastic constants. The prerequisite for effective strengthening of the brittle matrix by incorporating spherical particle of different elastic properties and a higher thermal expansion than the matrix, is that the particles have a higher strength and a diameter smaller than the pre-existing flaw size of the matrix. Thus, incorporation of high-strength particles in a low-strength matrix does not automatically lead to the expected strengthening of the resultant composite.

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[*] The modification is required here because fracture occurs from pores and not from inherently sharp cracks.